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The number of periodic solutions of a Riccati type logistic periodic equation

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Abstracts

In this paper, by using a simple transformation of variables and the necessary and sufficient condition in order that a solution of a periodic equation be periodic it is proved that the periodic logistic equation (of Riccati type) has exactly two periodic solutions.

Keywords:-A periodic equation no of solution of equation.

Introduction

In [1] the question about the existence of w-periodic solutions of the non - autonomous logistic equation of population growth

$$Y^{\bullet} = [r(t) - E(t)Y - \frac{r(t)}{k(t)} Y^2 \ldots(1)$$

where r(t), k(t) and E(t) are positive w-periodic functions was discussed. Equation (1) can be regarded as a model for the growth of a population Y(t) at time t which is being proportionally harvesting in a periodic manner. Function r(t) is the intrinsic growth rate and k(t) is the carrying capacity and E(t) is regarded as a measure of harvesting effort.

In the mentioned paper it was proved the existence of two w-periodic solutions of equation (1) by using a standard field and fixed point argument. Some bounds for the

non-constant w- periodic solutions were given too. In the same direction in [2] and in [3] the case of not harvesting, that is the case E(t) = 0 , was studied

It is worth mentioning that equation

$$Y^{\bullet} = Y^2 + \alpha(t)Y + \beta(t)$$

which was studied by Lloyd [4]is a special case of the equation being studied now . In 1999 , Miklaszewski[5]proved that the Ricatti equation has no 2π periodic solutions. Also in[1], Liao and yang gives some results about the growth and factorization of entire solutions of Riccati equation.

Here we extend these results to a more general Riccati equation by using a simple transformation of variables and the necessary and sufficient condition in order thata solution of the w- periodic Riccati equation be w- periodic.

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MAIN RESULTS

Let us consider the initial value problem (I.V.P)

$$Y^{\bullet} = a(t)Y + b(t)Y^2 \quad ....(2)$$
$$Y(0)=Y_0 , Y_0 \in R$$

where a(t) , b(t) : $[0,\infty) \to R$ are continuous functions , and let

$$\varphi(t) = \exp\left(\int_0^t a(\xi)d\xi\right) \quad ...(3)$$

Theorem 1: The solution of the I.V.P (2) is given by the function

$$Y(t,Y_0) = Y_0\left[1 - Y_0 \int_0^t b(\xi)\varphi(\xi)d\xi\right]^{-1}\varphi(t)$$

which is defined for all t such that

$$1 - Y_0 \int_0^t b(\xi)\varphi(\xi)d\xi \neq 0$$

Proof:

The transformation $Y= \varphi(t) X$ ...(5)

Reduces the equation (2) into the equation in separable variables

$$X^{\bullet} = b(t)\varphi(t)X^2 \quad ....(6)$$

From (3) and (5) it is clearly seen that

$Y(0) = X(0) = Y_0$

Therefore, after integrating (6) and using the inverse transformation of (5) we obtain function (4).

Remark: it is a well known fact that if a(t) is an w- periodic function and b(t)=0 , then the equation

$$Y^{\bullet} = a(t)Y$$

has a non -constant w-periodic solution if and only if $\int_0^w a(t)dt = 0$

Therefore, in the following we consider the case $b(t) \neq 0$,we also suppose that $a(t) \neq 0$

Before giving the main result of this paper we state the following theorem which is proved in [6].

Theorem 2 : Let r(t), k(t), E(t) : $[0, \infty) \to R$ be continuous positive w- periodic functions such that r(t) -E(t)> 0.

Then the w- periodic logistic equation (1) has exactly two w- periodic solutions, namely the zero solution and the one given by the function

$$t \in [0,\infty], where \psi(t) = \exp\left(\int_0^t [r(\xi) - E(\xi)]d\xi\right)$$

and $\delta(w) = 1 - \psi(w)$.

We are now ready to give the main result.

Theorem 3 :

Let a(t), b(t): $[0, \infty) \to R$ be respectively continuous non-negative and non -positive w-periodic functions. Then, the non - autonomous w- periodic Riccati equation (2) ,

$$Y(t) = \delta(w)\left[\int_0^w b(\xi)\phi(\xi)d\xi - \delta(w)\int_0^t b(\xi)\varphi(\xi)d\xi\right]^{-1}\varphi(t)$$

has exactly two w-periodic solutions. Namely, the zero solution and the one given by the function

......(9)

Which is defined for all $t \in [0, \infty)$ and where $\phi(t)$ is given by (3) and $\delta(w)=1-\varphi(w)$

Proof:

Under the stated hypothesis the non-Autonomous Riccati equation(2)is w-periodic, therefore a solution $Y(t,Y_0)$ of that equation as given in(4) is w-periodic if and only if

$$Y(0,Y_0) - Y(w,Y_0) = 0 \qquad \ldots\ldots\ldots\ldots(10)$$

By substituting function (4) in (10) we obtain

$$Y_0[1-(1-Y_0\int_0^w b(\xi)\varphi(\xi)d\xi)^{-1}\phi(w)]=0 \qquad \ldots\ldots\ldots\ldots\ldots (11)$$

Which allows us to determine the initial values in such a way that the corresponding solutions of equation (2) be w-periodic, obviously $Y_0=0$ satisfies equation (11) ,to this initial value the corresponding solution is by virtue of the theorem of existence and uniqueness, the zero solution Y(t), which is obviously an w-periodic function. Let us, now suppose that $Y_0 \neq 0$ in (11) , then from (11) we have that Y_0 is defined in a unique way by formula:

....(12)

It is clear from (12) that Y_0 is a positive number, since from the stated hypothesis the integral and $\delta(w)$ in (12) are negative numbers. Now by substituting (12) in (7) we obtain after some calculations the solution (9) , which is clearly defined for $t \in [0, \infty)$.

Remark:

By setting $a(t)= r(t)- E(t)$ and $b(t)=-r(t)/k(t)$. From theorems 1 and 3 we obtain theorem 2 (see [6]). In case of not harvesting, it is in case $E(t)=0$, theorem 2 is a result obtained in [1].

Example 1:

Let us consider the 2π- periodic logistic equation

$$Y^{\bullet}=Y-(2+\sin(t))Y^2 \qquad \ldots(13)$$

where $a(t)=1$, $b(t)= -(2+\sin(t))$.

In this case $\varphi(t)=e^t$ and the solution of (13) which satisfies the initial condition (2) is given by the function:

$$Y(t,Y_0)=[1+Y_0(e^t(4+\sin t-\cos t)/2-3)]^{-1}Y_0e^t$$

Now , equation (11) gives us the values:-

$Y_0=0$ and $Y_0=2/3$.

Example 2:

Let us consider the 2π periodic logistic equation

$$Y^{\bullet}=(2+\sin t)Y-e^{-1}Y^2$$

where $a(t)=2+\sin t$, $b(t)=-e^{-1}$. In this case $\varphi(t)=\exp(2t+1-\cos(t))$ and the solution of (14), which satisfies the initial condition (2) is given by the function:

$$Y(t,Y_0)=Y_0[1+Y_0\int_0^t e^{2\zeta-\cos\zeta}d\zeta]^{-1}e^{2t+1-\cos t}$$

Now, equation (11) gives us the values For which the corresponding 2π periodic solutions are respectively, the zero solution Y(

$$Y_0 = 0 \quad and Y_0 = \frac{e^{4\pi} - 1}{\int_0^{2\pi} e^{2\pi - \cos(t)} dt} \quad ...(15)$$

Where $H(t) = (e^{4\pi} - 1)e^{2t+1-c0s(t)}$

In this example the value of Y_0 corresponding to the non- constant 2π periodic solution (16)

$$Y(t, Y_0) = [\int_0^{2\pi} e^{2t-\cos(t)} dt + (e^{4\pi} - 1)\int_0^{t} e^{2\xi-\cos(\xi)} d.$$

has to be calculated via numerical integration which leads to an approximated value to Y_0 .We calculated the value of the integral in (15) by using Sympsons rule and obtained the approximate value Y_0= 4.1232706.

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ژمارەی شیکارەخولەییەکان بۆ هاوکێشەی ریکاتی بەکارهێنراو لە بۆ ماوە

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پوختە

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عدد الحلول الدورية لمعادلة ريكاتى الدورية المستخدمة في الوراثة

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الخلاصة

هذا البحث الشرط الضروري والكافي لمعادلة ريكاتي الدورية وباستخدام تحويل المتغيرات ، أن يكون لها حلا دوريا وبالتحديد حلين دوريين.

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